

Ersatzmodelle in konvektionsdominierten Störungssystemen: Erkenntnisse für zuverlässige Umsetzungen

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Unsicherheiten in der Geothermie

- > Sensitivitätsanalysen
- > Unsicherheitsanalysen

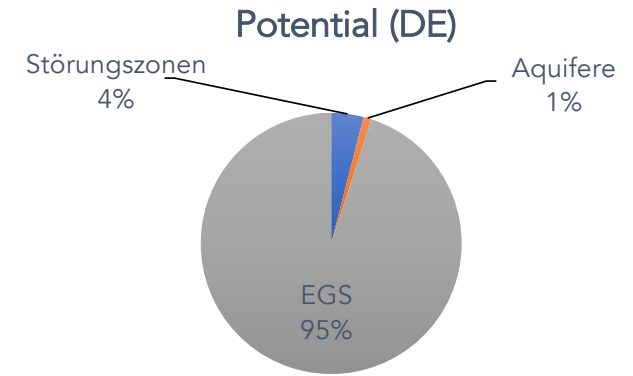
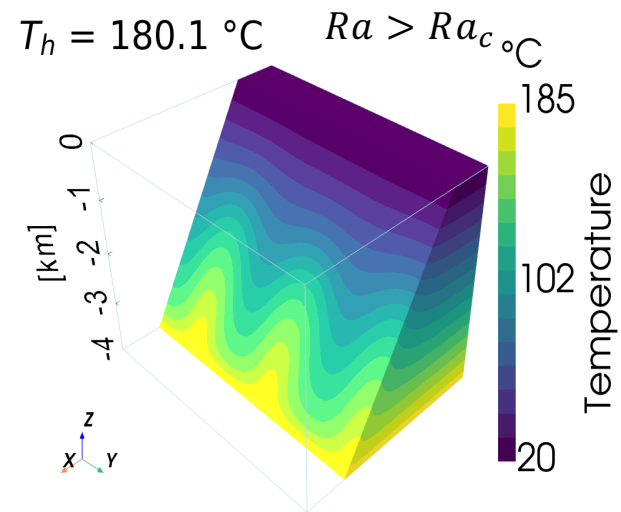
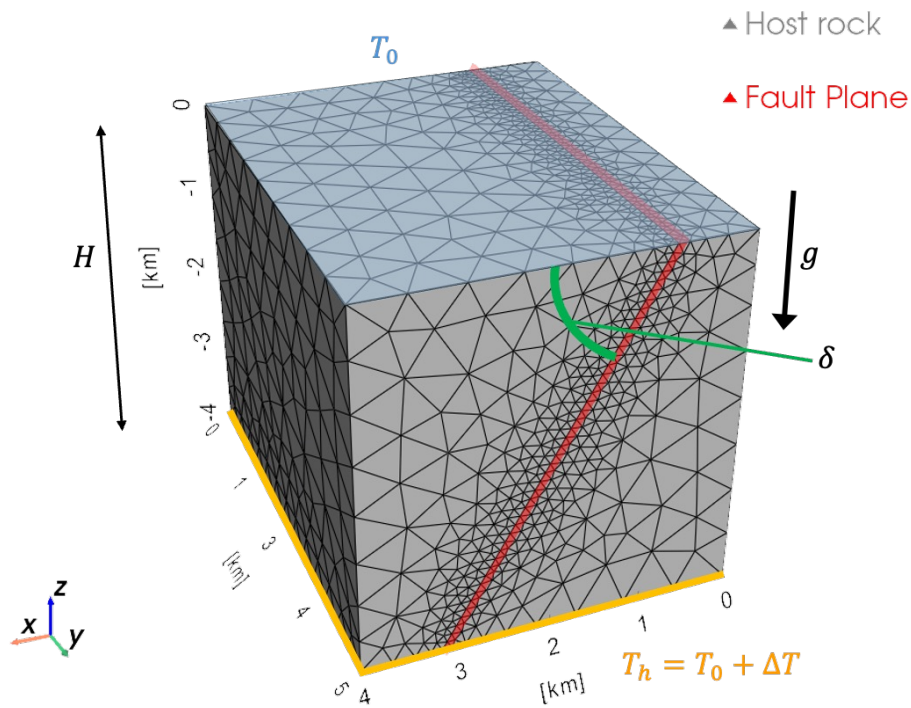
Rechenintensiv



Ersatzmodelle können die Simulationszeit signifikant beschleunigen.



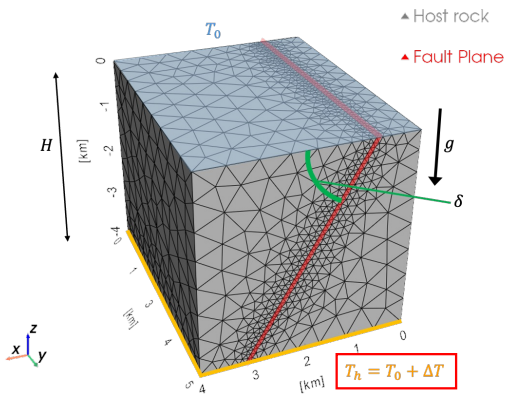
Störungszonen können positive Temperaturanomalien aufweisen



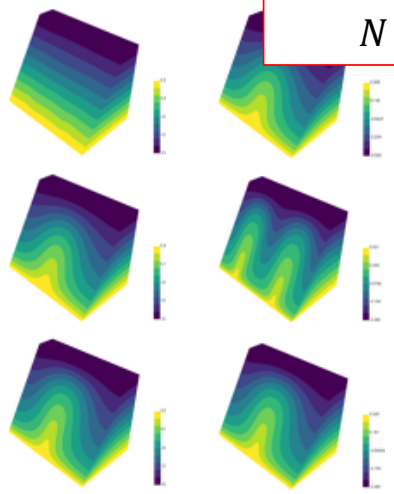
Jung et al. 2002, S. 78

Erstellung von Ersatzmodellen mit physikbasierten Machine Learning

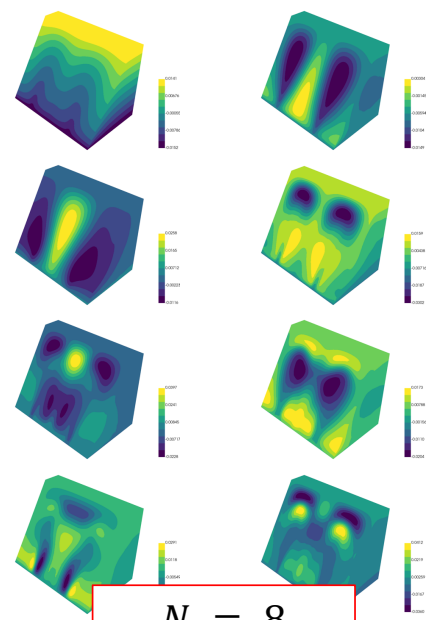
Non-intrusive reduced basis method (NI-RB)



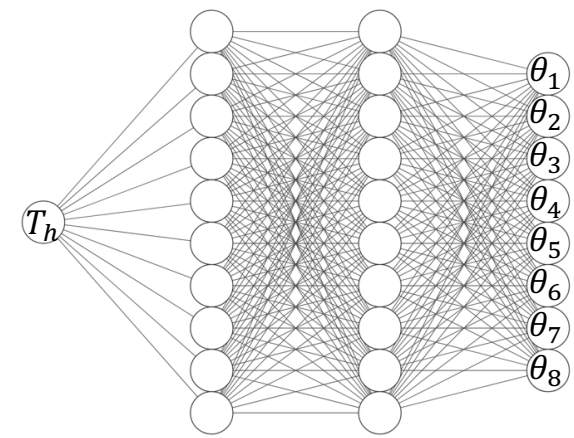
Trainingsdaten
 $N = 100$



Proper Orthogonal Decomposition (POD)
-> Basisfunktionen ψ

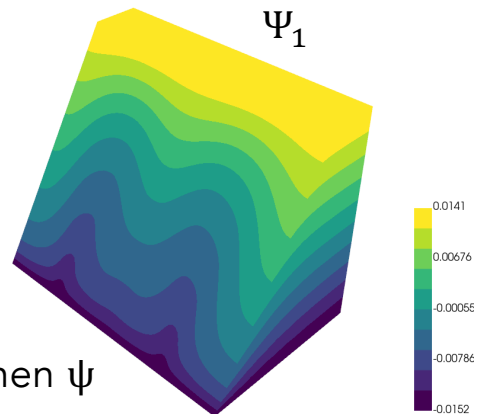


Neuronales Netzwerk
-> Koeffizienten θ

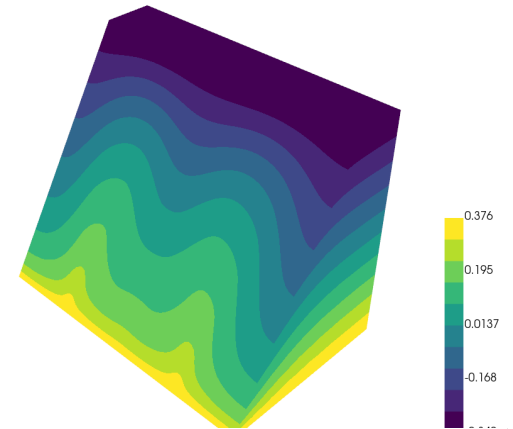
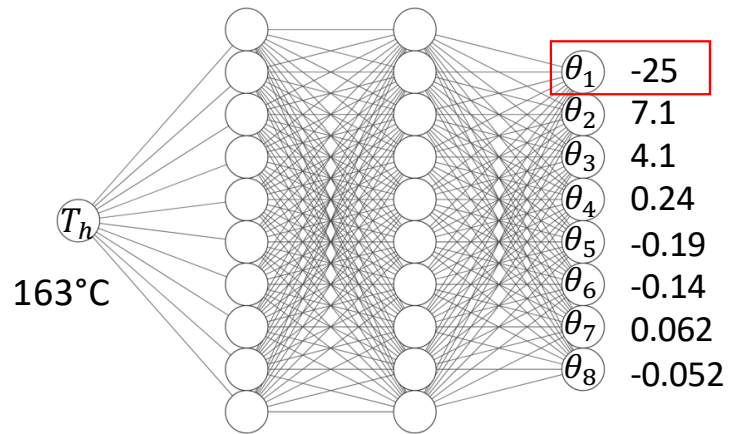
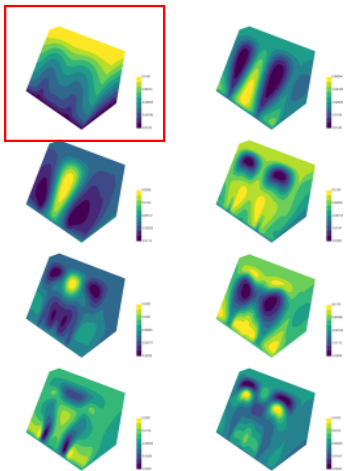


Ersatzmodell
 $\sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$

Beispiel: $T_h = 163^\circ\text{C}$

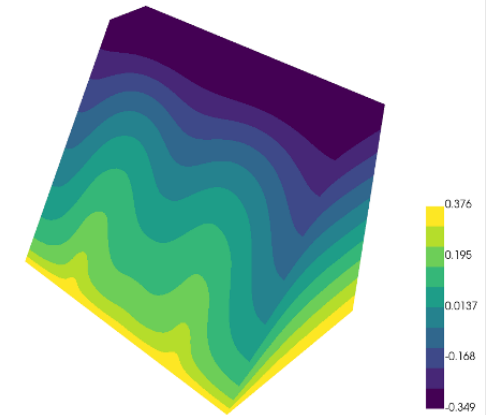


Basisfunktionen ψ



Ersatzmodell:

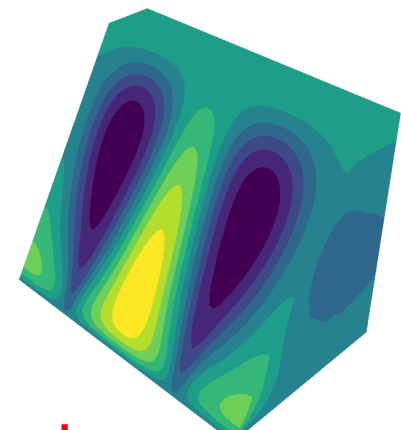
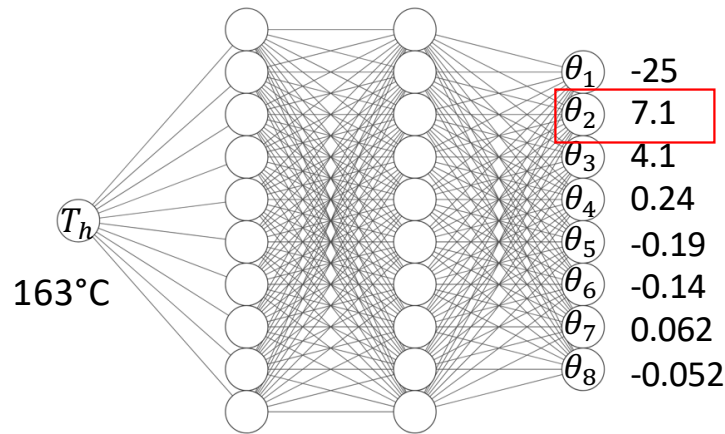
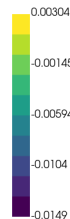
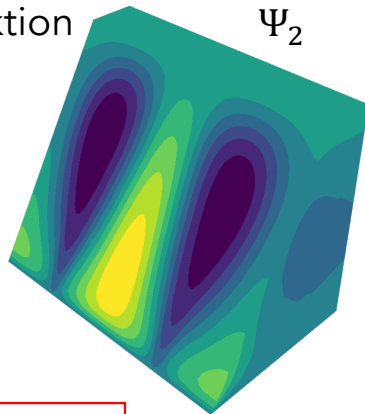
$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



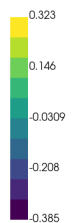
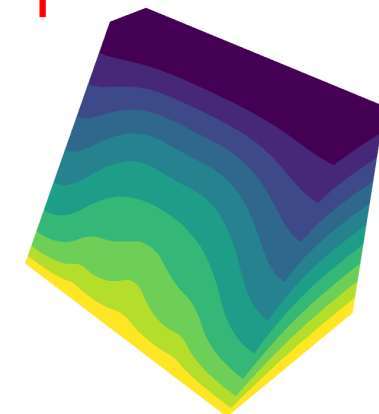
Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

Ψ_2

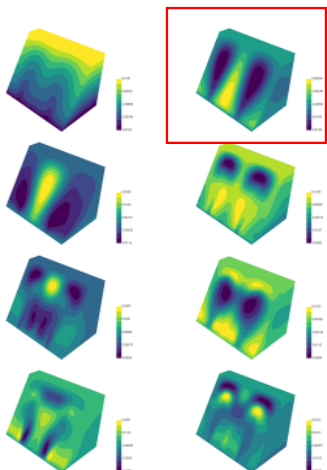


+



Ersatzmodell:

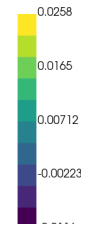
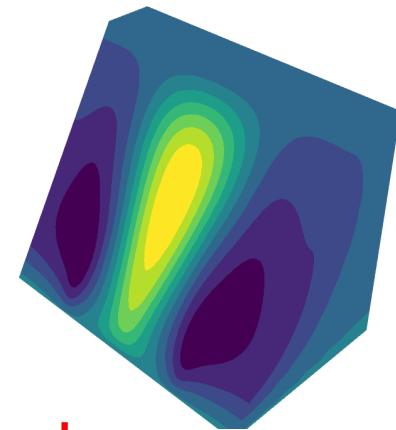
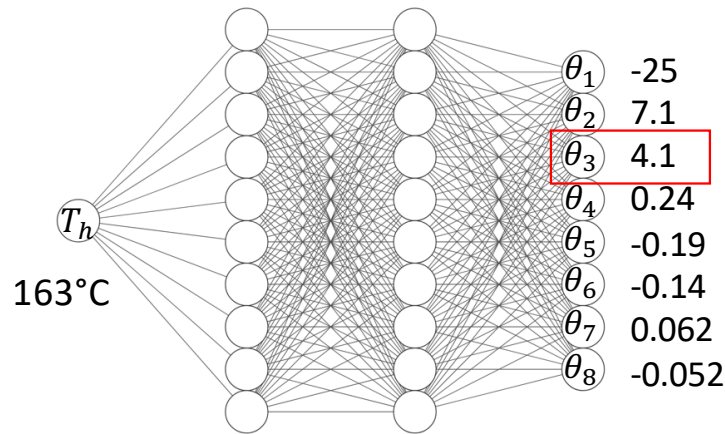
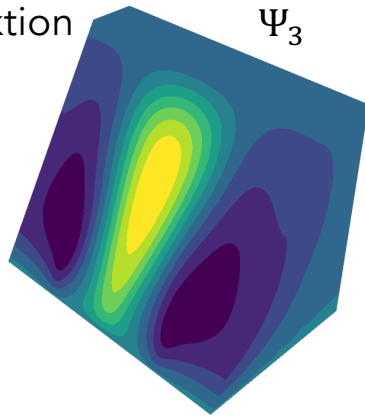
$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



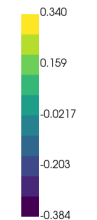
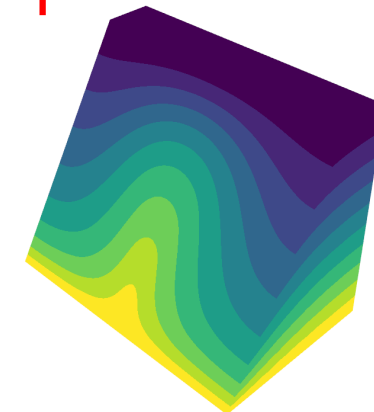
Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

Ψ_3

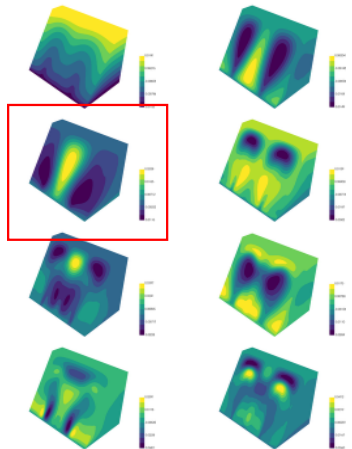


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Ersatzmodell:

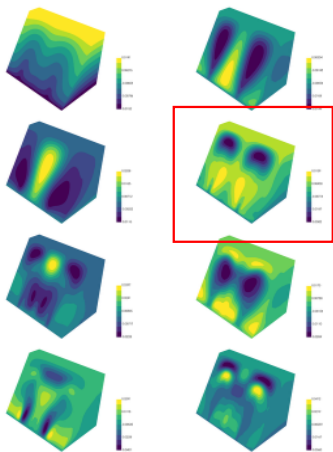
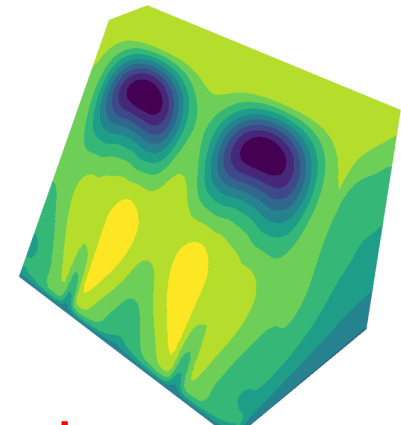
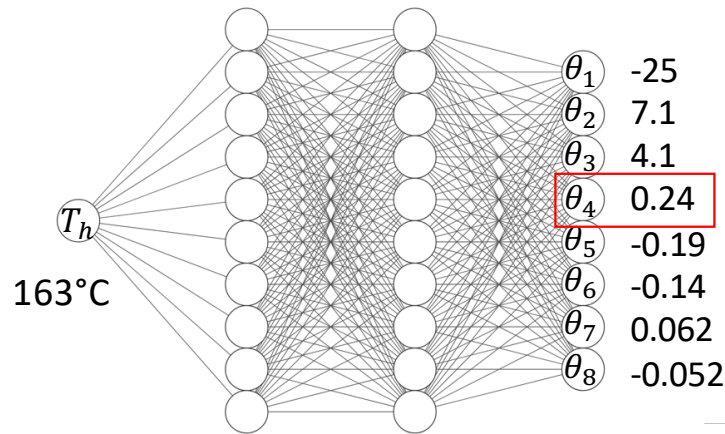
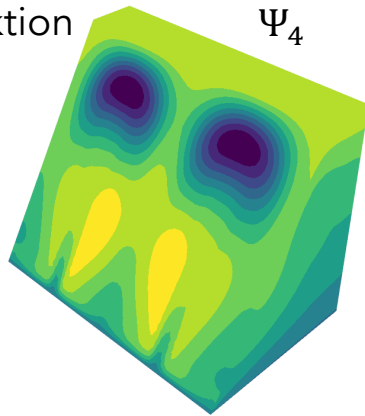
$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

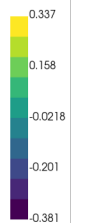
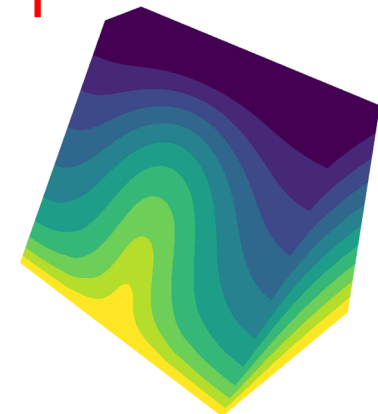
Ψ_4



Ersatzmodell:

$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$

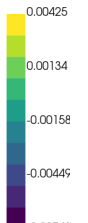
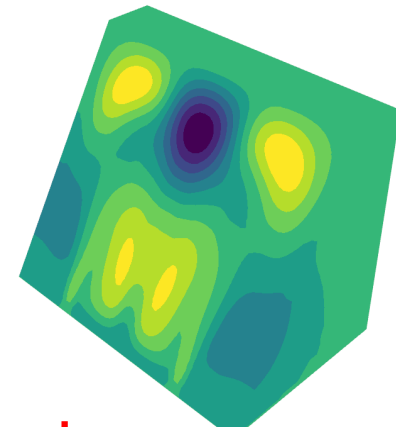
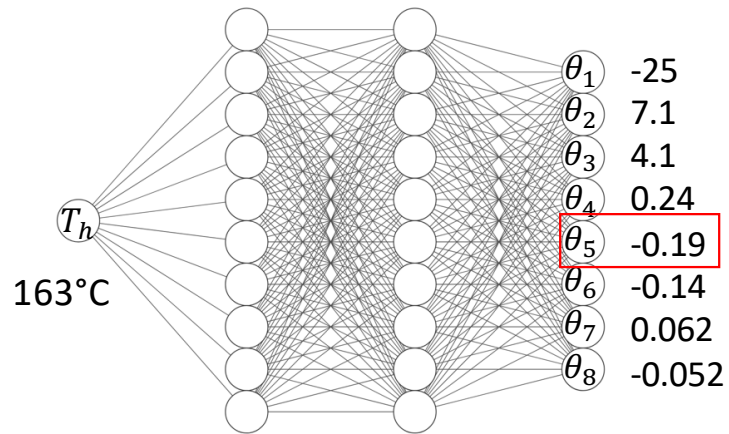
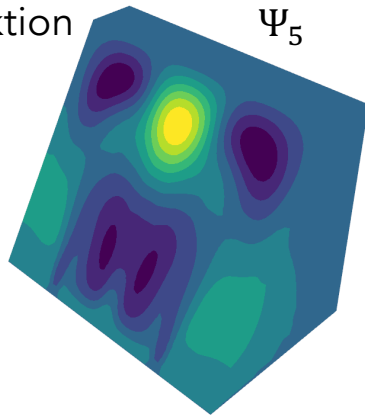
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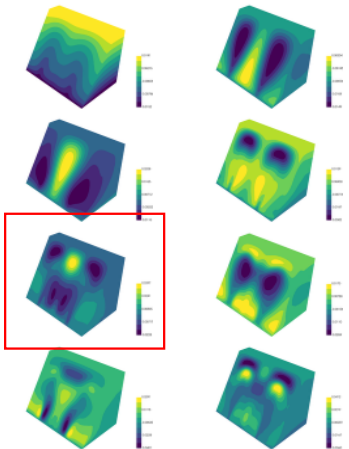
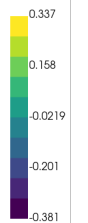
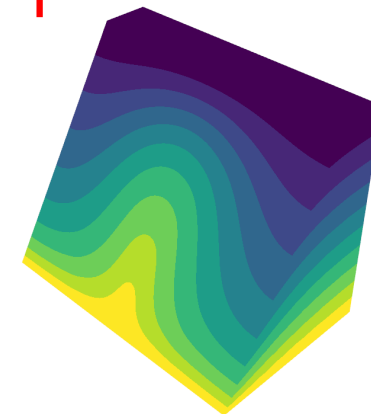
Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

Ψ_5



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Reduced Order Model

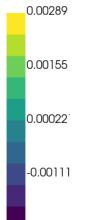
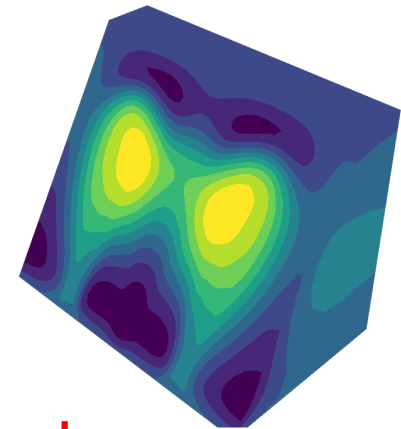
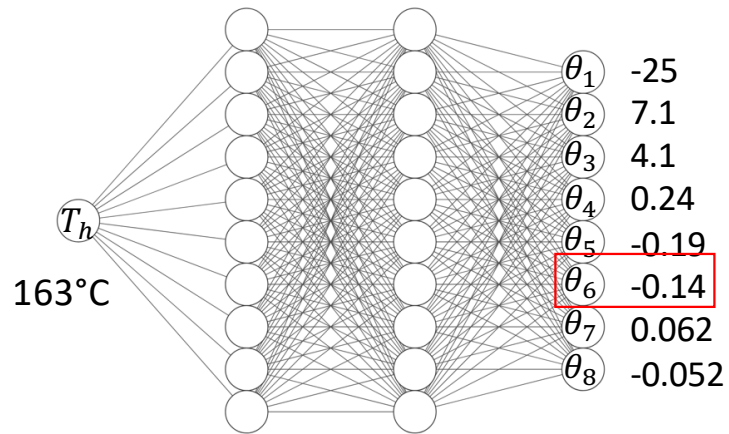
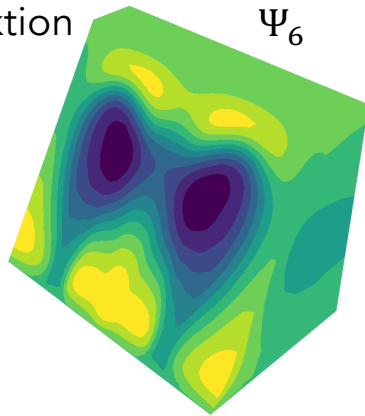
$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



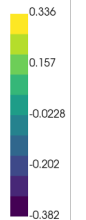
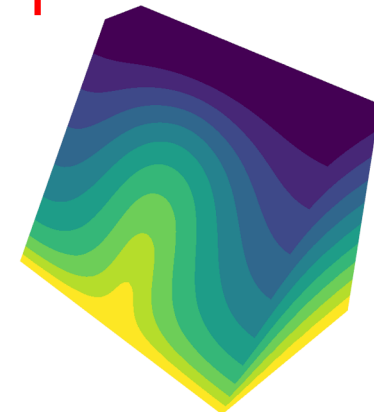
Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

Ψ_6

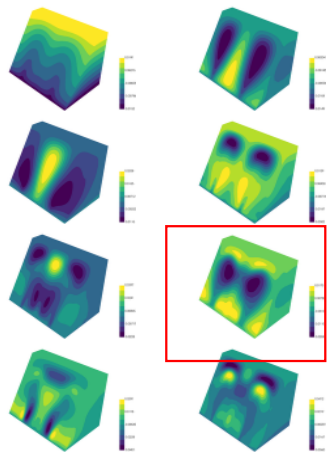


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Ersatzmodell:

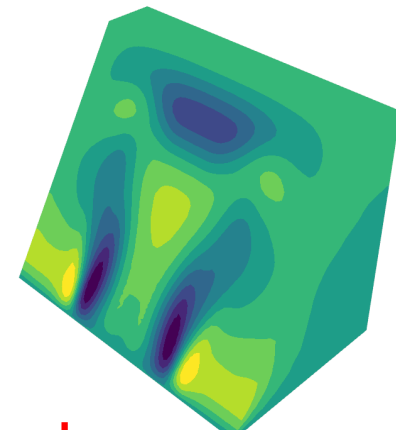
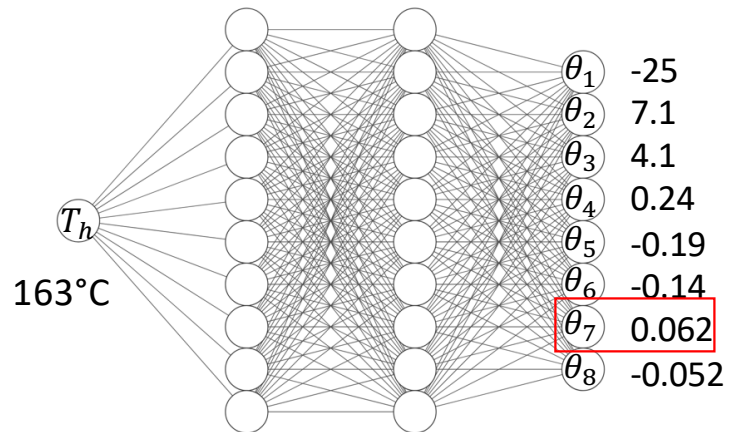
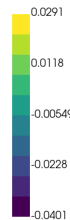
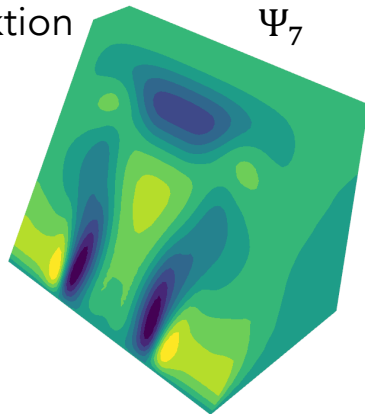
$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



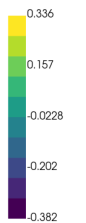
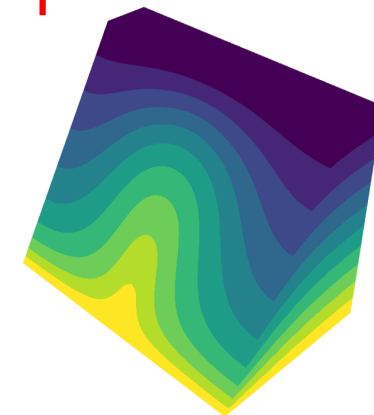
Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

Ψ_7

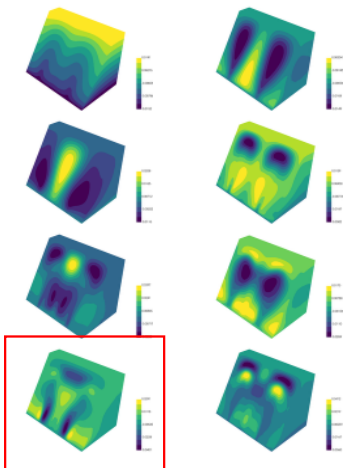


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Ersatzmodell:

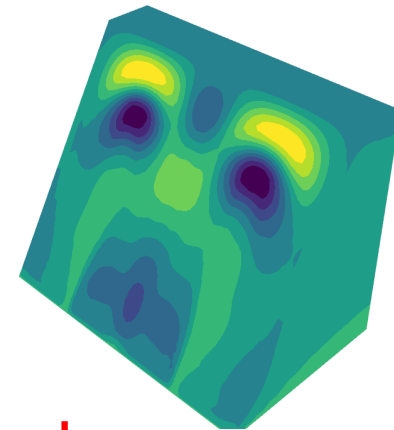
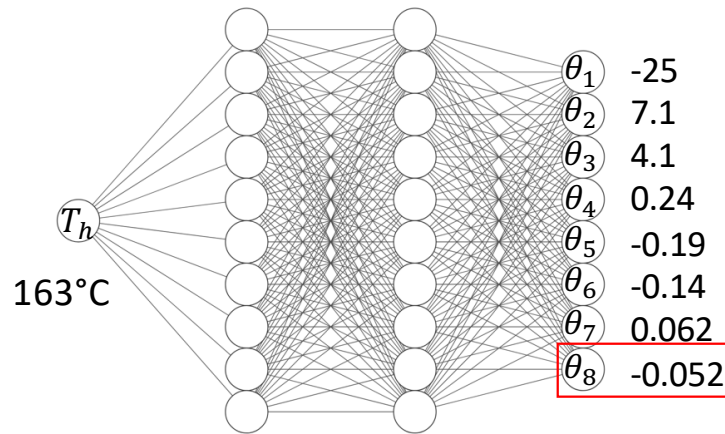
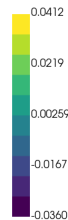
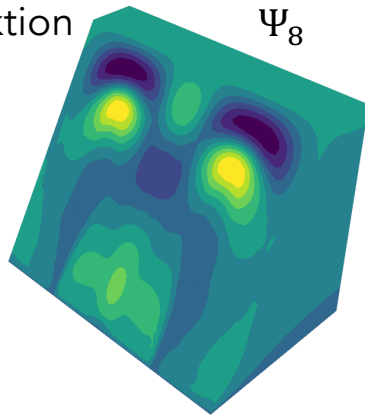
$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



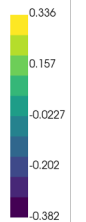
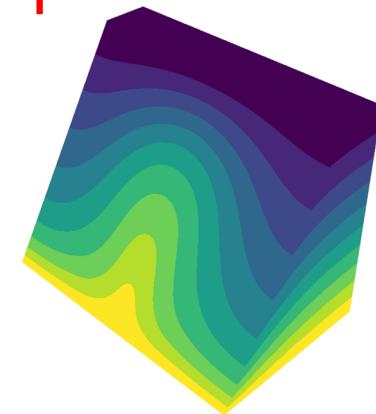
Beispiel: $T_h = 163^\circ\text{C}$

Basisfunktion

Ψ_8

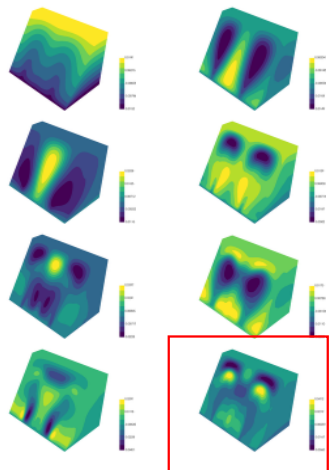


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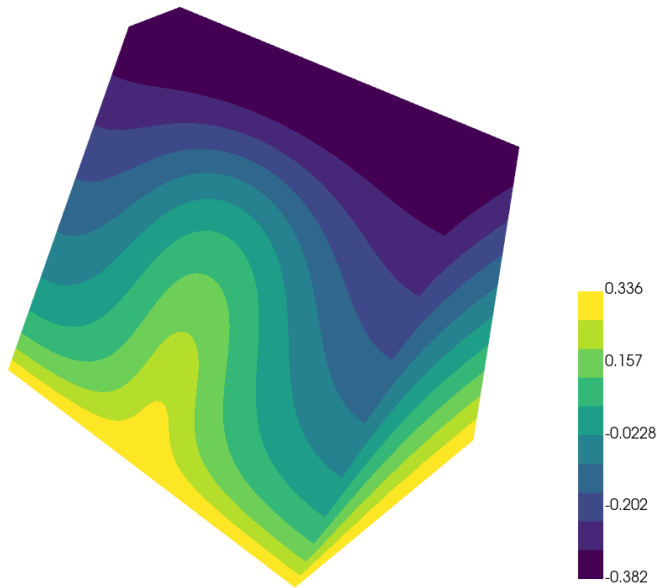
Ersatzmodell:

$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



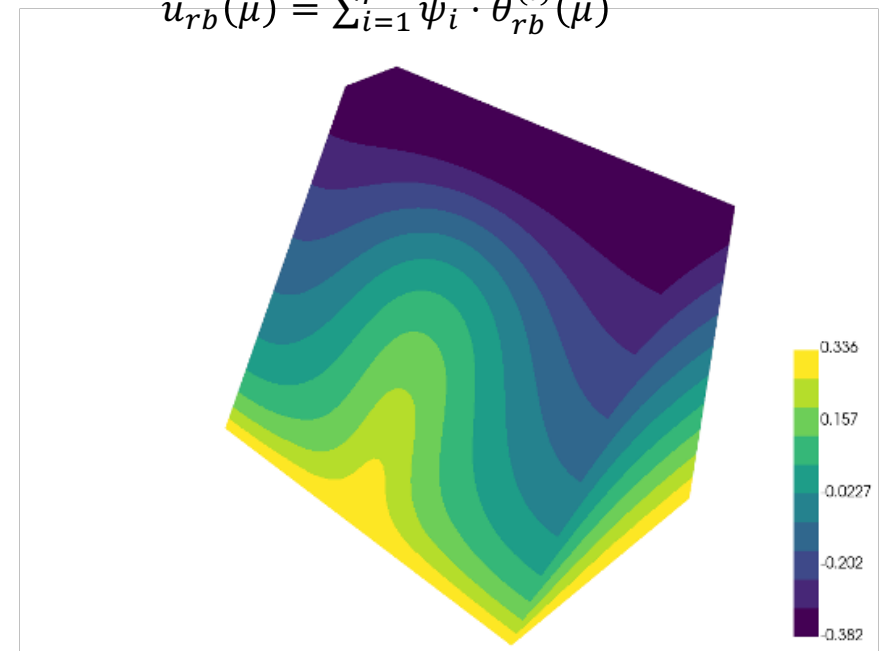
Beispiel: $T_h = 163^\circ\text{C}$ – Vergleich

Numerische Lösung



Ersatzmodell

$$u_{rb}(\mu) = \sum_{i=1}^r \psi_i \cdot \theta_{rb}^{(i)}(\mu)$$



Die Entropieproduktion eignet sich zur Charakterisierung hydrothermaler Systeme

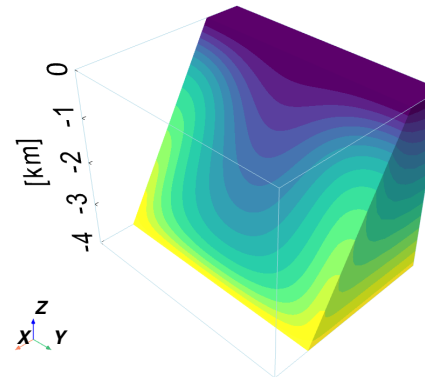
> Thermische Entropieproduktion [WK⁻¹s⁻¹m⁻³]:

$$S'''_{gen} = \frac{\lambda}{T_0^2} (\nabla T)^2.$$

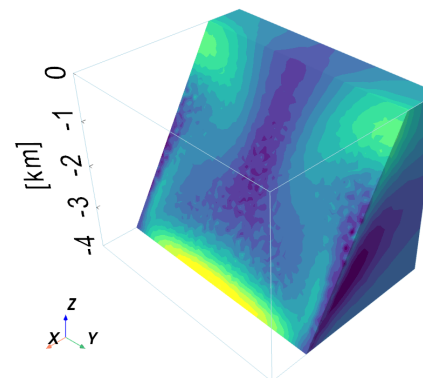
> Entropieproduktionszahl [-]:

$$N_s = \int S'''_{gen} dV \cdot \frac{H^2 T_0^2 V}{\lambda (\Delta T)^2}.$$

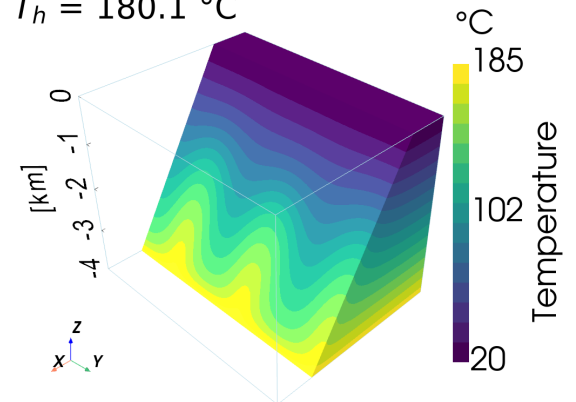
$T_h = 178.3 \text{ }^\circ\text{C}$



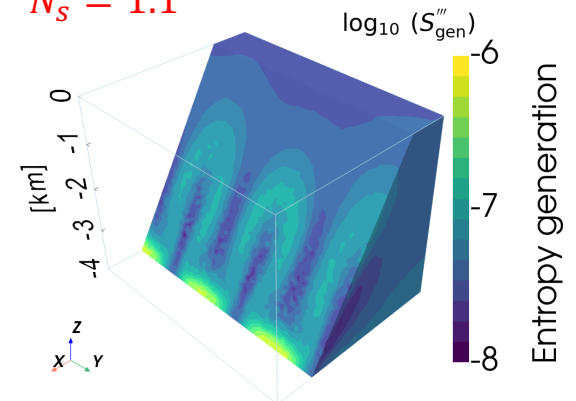
$N_s = 1.3$



$T_h = 180.1 \text{ }^\circ\text{C}$

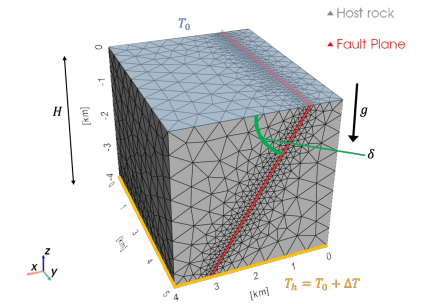
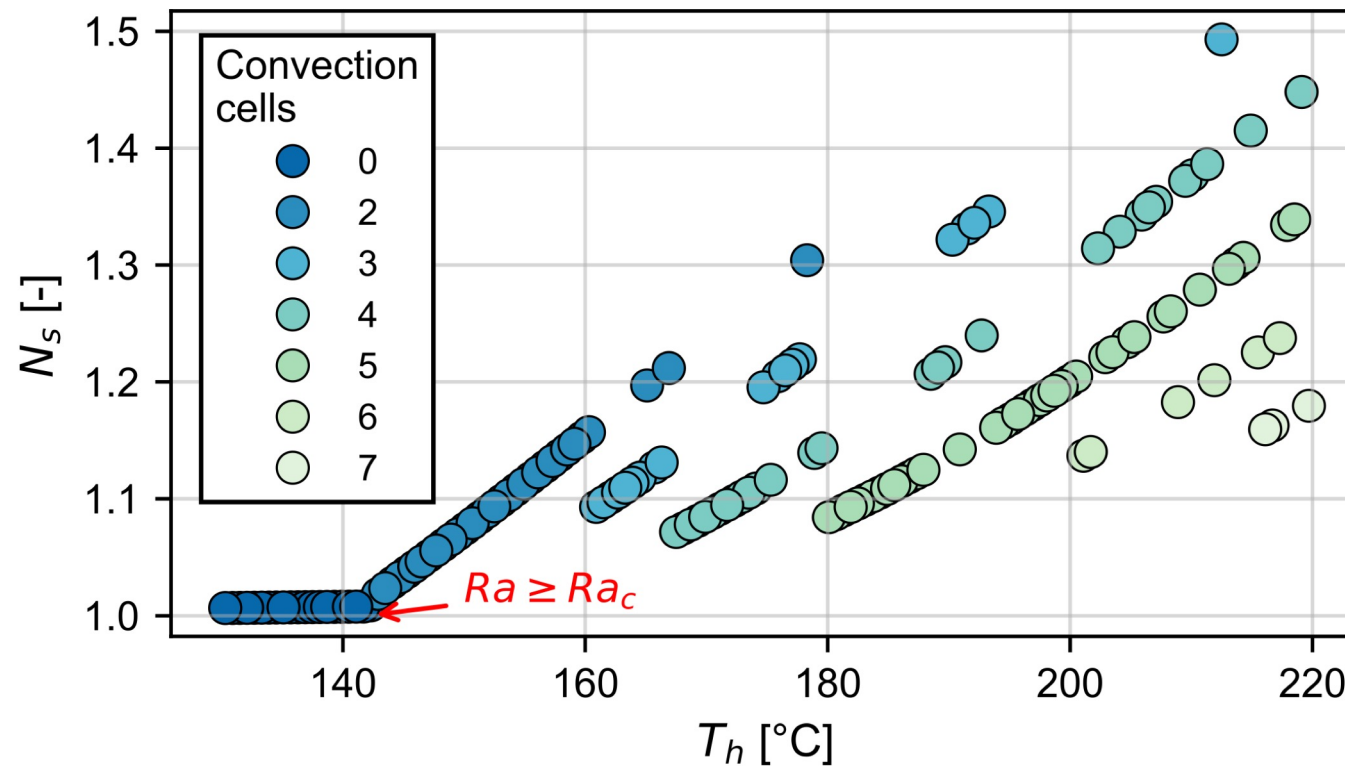


$N_s = 1.1$

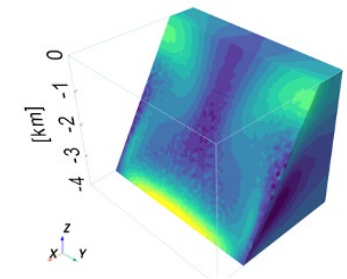


Physikalische Komplexität: Multistabilität

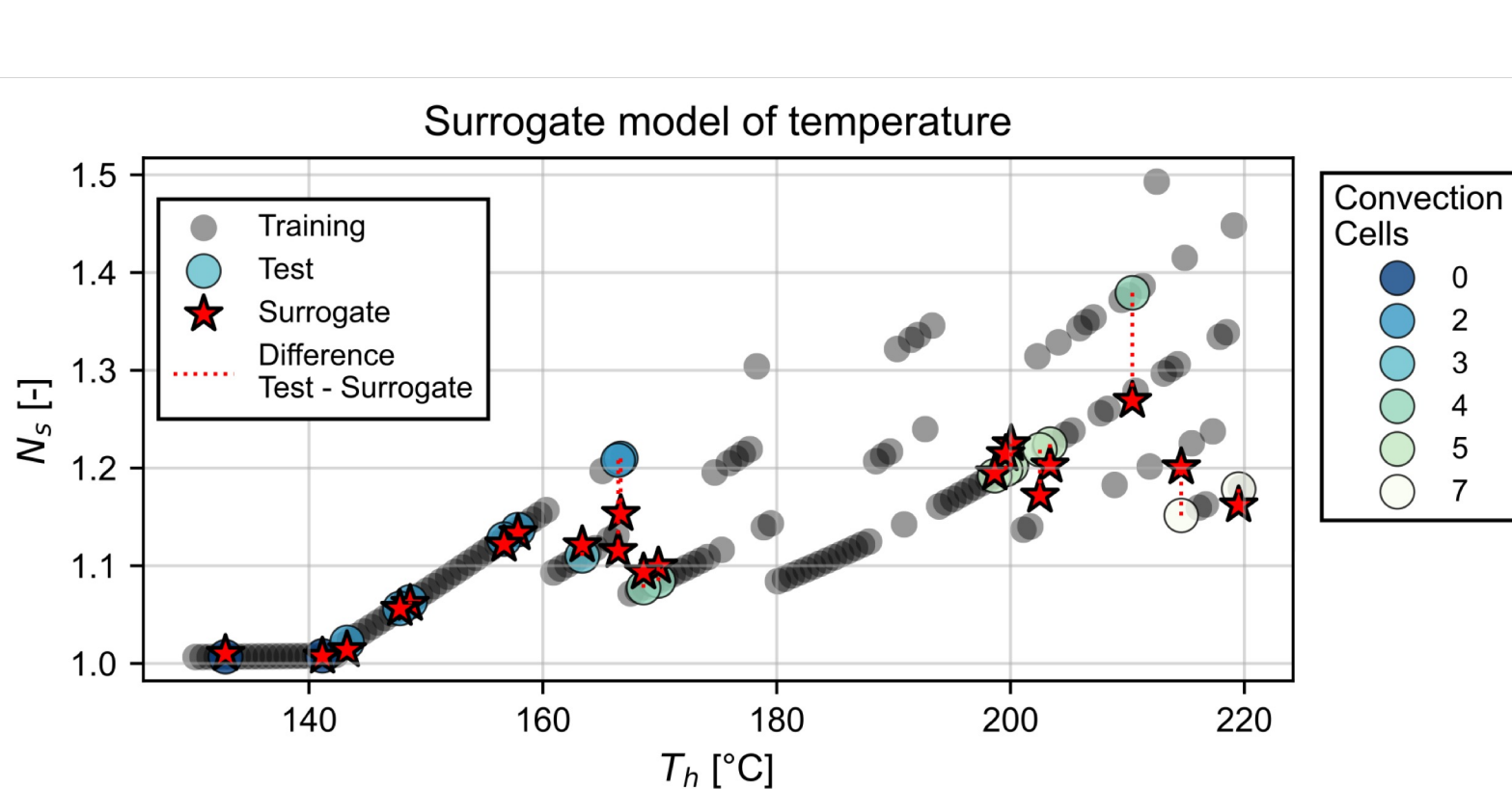
Training data when varying T_h



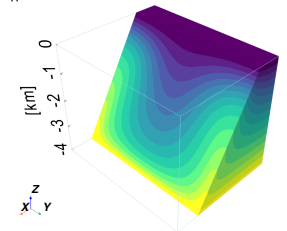
$N_s = 1.3$



Performance eines temperaturbasierten Ersatzmodells

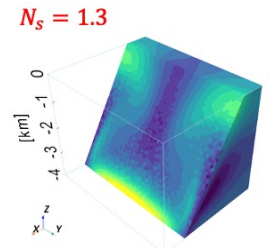
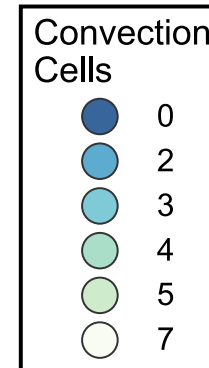
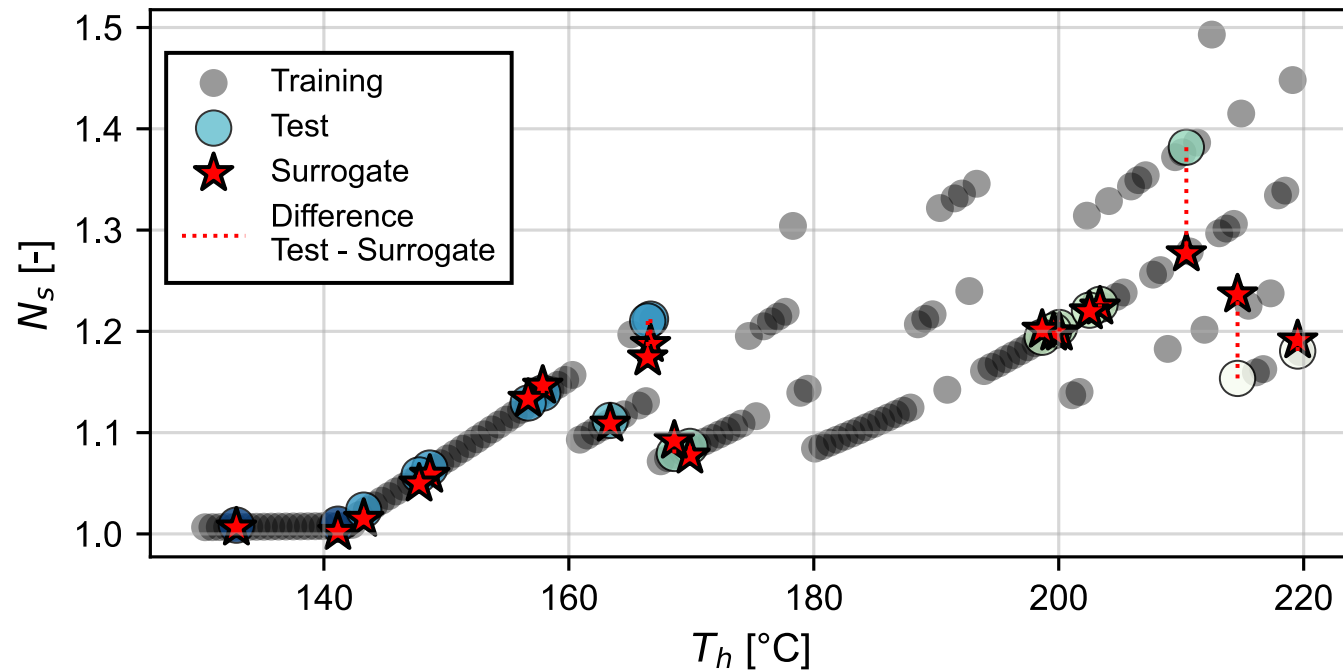


$T_h = 178.3$ °C



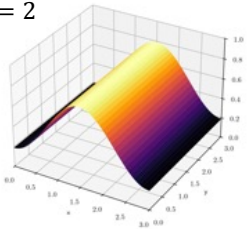
Entropiebasierte Ersatzmodelle liefern bessere Ergebnisse

Surrogate model of entropy generation

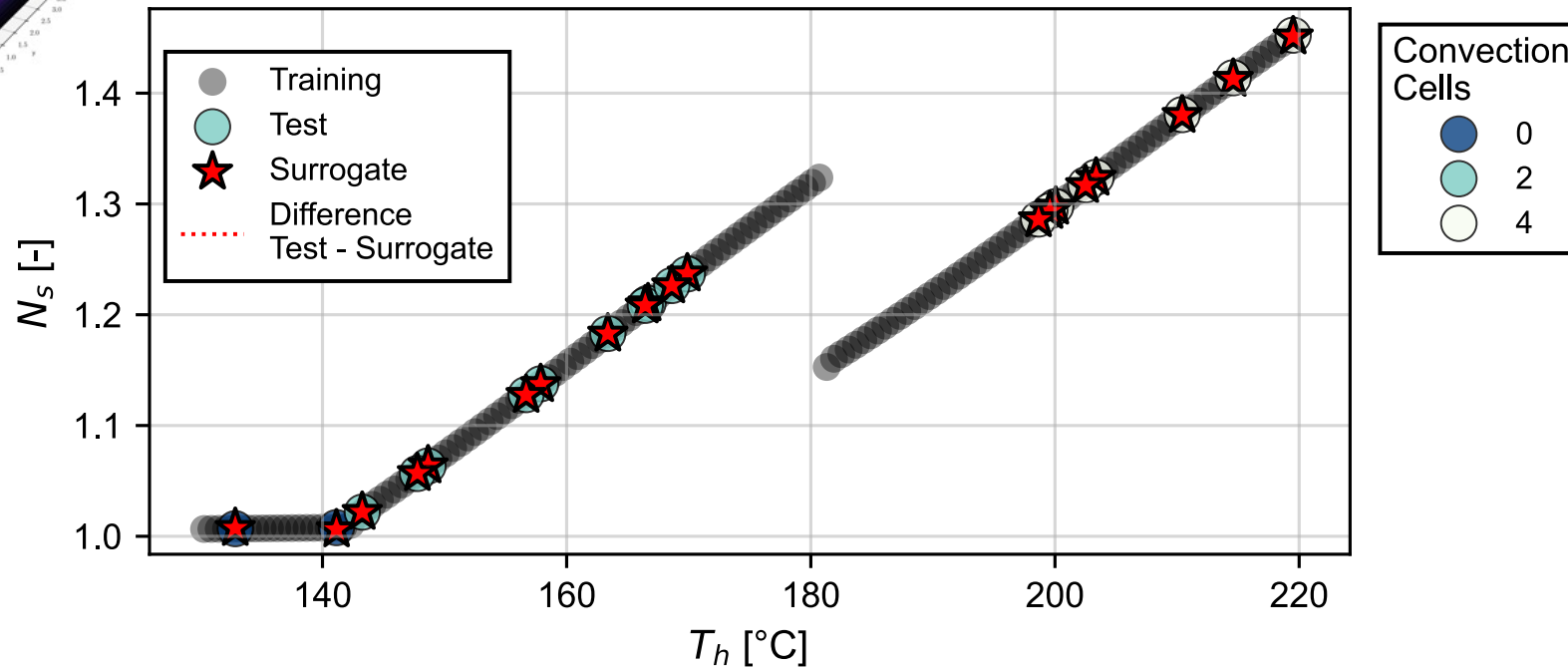


Verbesserung durch geeignete Wahl der Temperatur-Anfangsbedingung

$n = 2$



Surrogate model of entropy generation



N_s eignet sich als Zielgröße für stochastische Analysen

Beispiel: Sensitivitätsanalyse

1. Vorbereitungsschritt für eine Modellkalibrierung
2. Verbesserung des Modellverständnisses durch Ableitung dominanter Prozesse.

	Numerische Modellierung
150 Trainingsdaten	-
20 Testdaten	-
Neuronales Netzwerk	-
Sensitivitätsanalyse (2 Mio Evaluierungen)	57 Jahre
Summe	57 Jahre

1 numerischer Durchlauf \approx 15 Min



Take-home messages

1. Störungzonen können geothermische Potentiale aufweisen.
2. Ersatzmodelle lassen sich für konvektive Systeme erstellen (ggf. durch geeignete Anfangsbedingungen)
3. Die Entropieproduktionszahl N_s eignet sich für die Charakterisierung hydrothermalen Systeme und als Zielgröße für einhergehende Analysen.
4. Entropiebasierte Ersatzmodelle können eine höhere Genauigkeit erreichen als temperaturbasierte Ersatzmodelle.



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